Epsilon Problem

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Students per Grade 2022-2023

Department	10th	11th	12th	Total
Art	31	33	35	99
Biology	198	95	26	319
Chemistry	59	126	109	294
English	183	155	152	490
French	41	32	49	122
German	19	22	10	51
Spanish	51	26	33	110
Mathematics	184	201	262	647
Music	50	56	49	155
Physics	50	58	183	291
Social Studies	183	131	59	373

1. We noticed that the total number of students in English class is 490 which is equal to the total number of students.

2. We then assumed that every student takes one English class.

3. From there, we concluded that the number of students in English class for each year is the number of students in the corresponding grade

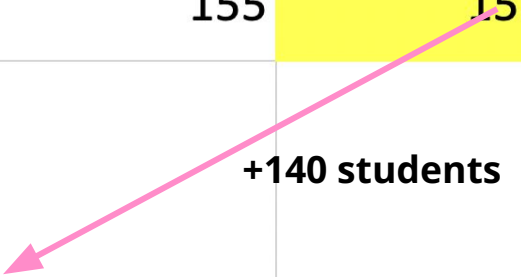
Four English units are required in Massachusetts high schools, with one course taken each year.

Contents of this problem

- A school with 490 students during 2022-2023
- Up to 630 students 2023-2024
- Normally, the # of graduating seniors = # of incoming sophomores
  - (plus any dropouts)
- Next year, however, there will be 140 more sophomores than usual
- Seven new teachers will be hired to make up for new students
- We need to decide which classes should get the new teachers

How to calculate number of incoming Sophomores

	10th	11th	12th	Total
2022-2023	183	155	152	490
2023-2024	292	???	???	630?

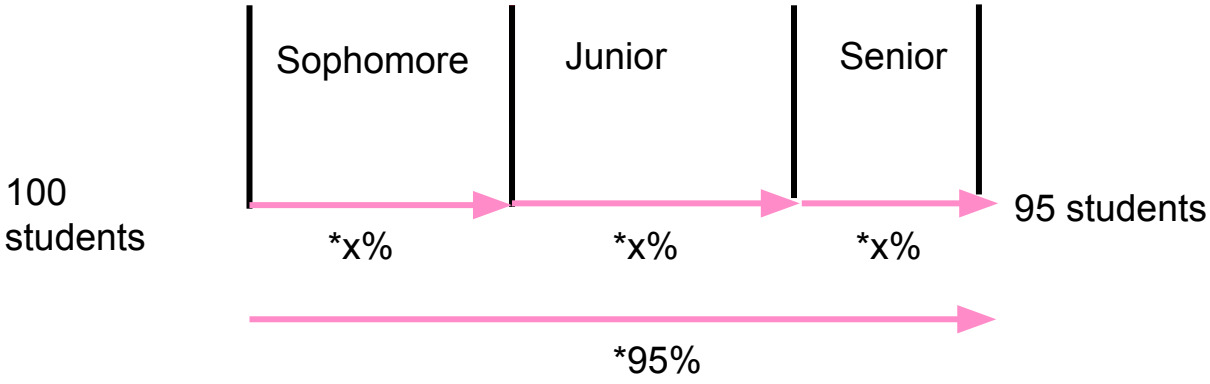


The incoming Sophomore class (the current Freshman class) will have “140 more students than the graduating senior class.” We thus concluded that the incoming Sophomore class will have:

152 + 140 = 292 students

Factoring in Drop-out rates

We assumed that the percentage of students dropping out each year would be equivalent. From this assumption we could do the calculations shown to the right.



$$100*x*x*x = 100*x^3 = 100*0.95$$
$$x^3 = 0.95$$
$$x=(0.95)^{⅓}$$

Number of Juniors and Seniors 2023-2024

	10th	11th	12th	Total
2022-2023	183	155	152	490
	Multiply by $(0.95)^{\frac{1}{3}}$		Multiply by $(0.95)^{\frac{1}{3}}$	
2023-2024	292	180	152	624

The problem description states: '**can** be increased from 490 to 630,' thus we assumed obtaining this value is not a necessity

To calculate the number of juniors and seniors during the 2023-2024 school year while taking into drop-outs, we multiplied the 2022-2023 class sizes by $(0.95)^{\frac{1}{3}}$ and rounded to the nearest 'whole' person.

Percentage of students in a grade that are in a specific class

We're assuming that for each grade, the percentages of students in each class are roughly the same every year.

# of students in each grade

Year	10th	11th	12th	Total
2023	183	155	152	490
2024	292	180	152	624

Department	10th	11th	12th	Total
Art	31	33	35	99

# of students in each class

31/183 = 0.1694...

	Grade Class Percentages per total class size		
Department	10th	11th	12th
Art	0.16939891	0.21290323	0.23026316
Biology	1.08196721	0.61290323	0.17105263
Chemisty	0.32240437	0.81290323	0.71710526
English	1	1	1
French	0.22404372	0.20645161	0.32236842
German	0.10382514	0.14193548	0.06578947
Spanish	0.27868852	0.16774194	0.21710526
Mathematics	1.00546448	1.29677419	1.72368421
Music	0.27322404	0.36129032	0.32236842
Physics	0.27322404	0.37419355	1.20394737
Social Studies	1	0.84516129	0.38815789
Total	5.73224044	6.01935484	6.36184211

of students in each class 2023-2024 (Using art as example)

% of students in a grade that are in a specific class

Grade Class Percentages per total class size			
Department	10th	11th	12th
Art	0.16939891	0.21290323	0.23026316

0.1694...*292 = ~49

# of students in each grade 2022-2023 & 2023-2024

Epsilon School Grade Enrollment Quantities				
Year	10th	11th	12th	Total
2023	183	155	152	490
2024	292	180	152	624

Epsilon School 2023-2024 Class Enrollment Quantities				
Department	10th	11th	12th	Total
Art	49	38	35	122

# of students in each class 2023-2024

of students in each class 2022-2023 (All Classes)

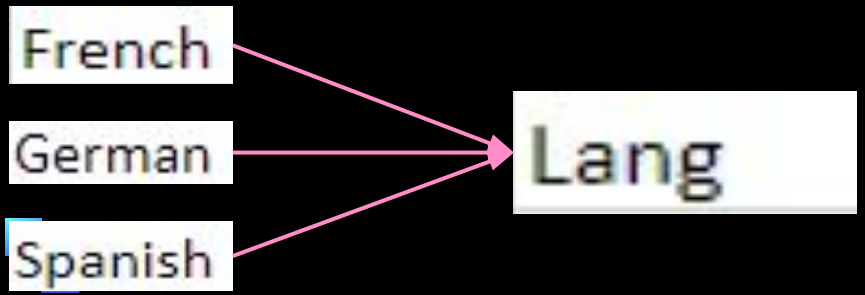
Epsilon School 2022-2023 Class Enrollment Quantities				
Department	10th	11th	12th	Total
Art	31	33	35	99
Biology	198	95	26	319
Chemisty	59	126	109	294
English	183	155	152	490
French	41	32	49	122
German	19	22	10	51
Spanish	51	26	33	110
Mathematics	184	201	262	647
Music	50	56	49	155
Physics	50	58	183	291
Social Studies	183	131	59	373
Total	1049	933	967	2949

of students in each class 2023-2024 (All Classes)

Epsilon School 2023-2024 Class Enrollment Quantities				
Department	10th	11th	12th	Total
Art	49	38	35	122
Biology	316	110	26	452
Chemisty	94	146	109	349
English	292	180	152	624
Lang	176	93	92	361
French	65	37	49	151
German	30	26	10	66
Spanish	81	30	33	144
Mathematics	294	233	262	789
Music	80	65	49	194
Physics	80	67	183	330
Social Studies	292	152	59	503
Total	1674	1083	967	3724

Combining Language Classes

For Spanish, French, and German, we assume the three Lang teachers are shared among the classes, thus they can be treated as a single ‘Lang’ subject until we determine which departments require teachers



	10th	11th	12th	Total
French	65	37	49	151
German	30	26	10	66
Spanish	81	30	33	144
Lang	176	93	92	361

2022-2023 Student-Teacher Ratios

Epsilon School 2022-2023 Student Teacher Ratios			
Department	Total	Number of Teachers	Stud:Teach Ratio
Art	99	1	99
Biology	319	4	79.75
Chemisty	294	3	98
English	490	5	98
Lang	283	3	94.33333333
Mathematics	647	6	107.8333333
Music	155	1	155
Physics	291	3	97
Social Studies	373	5	74.6

Given the number of students and teachers during the 2022-2023 school year, we calculated the student-teacher ratio. We assume there was prior planning relating to these values, thus we hope to parallel the student-teacher ratios for the 2023-2024 school year.

2023-2024 (Current) Student-Teacher Ratios

Epsilon School 2023-2024 Student Teacher Ratios			
Department	Total Students	Number of Teachers	Student:Teach Ratio
Art	122	1	122
Biology	425	4	106.25
Chemisty	349	3	116.3333333
English	624	5	124.8
Lang	361	3	120.3333333
Mathematics	789	6	131.5
Music	194	1	194
Physics	330	3	110
Social Studie	503	5	100.6

Solution #1: Gradient Descent

$$C = \sum_{i=1}^{n} (R_{b_i} - R_{c_i})^2$$

C = 'cost'

n = number of subjects/departments

R_b = Ideal student-teacher ratio for subject

R_c = Current student-teacher ratio for subject

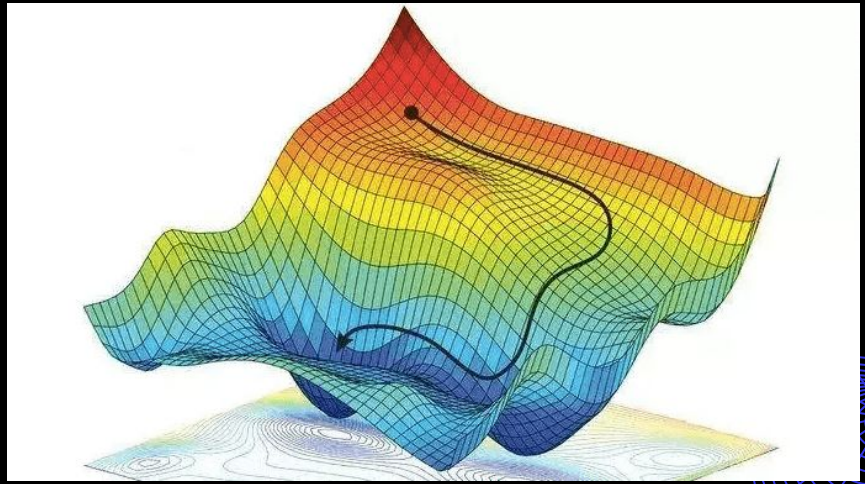
Rough Pseudocode:

1. Begin with zero 'added teachers'
2. Calculate 2023-2024 student-teacher ratio & independently add one to each 'added teacher' value
3. Calculate cost given added teacher in relation to the ideal ratio for 2022-2023 school year
4. Identify position in which hired teacher = minimized cost
5. Add teacher to position
6. Repeat steps 2-7 until all available teachers are hired

Solution #1: Gradient Descent (Pt. 1.5)

Why gradient descent?

- Used within many computational neural networks
- Minimizes cost by altering parameters - rate of steepest decrease
  - Teacher number to obtain near-ideal ratio
- Not data/memory intensive, takes short period of time, and easy to implement
  - Have pre-established equations for ratio & cost



Solution #1: Gradient Descent (Pt2)

```
enrollment = np.array([122, 452, 349, 624, 361, 789, 194, 330, 503])
idealRatio = np.array([99, 79.75, 98, 98, 283/3, 647/6, 155, 97, 74.6])
currentTeacherNum = np.array([1, 4, 3, 5, 3, 6, 1, 3, 5])

def getRatio(teachers, studs=enrollment):
    return studs/teachers


def cost(ratio, ideal=idealRatio):
    sum = 0
    for i in range(0, len(ratio)):
        sum += (ratio[i]-ideal[i])**2
    return sum

teacherNum = np.array([1, 4, 3, 5, 3, 6, 1, 3, 5])
teachersRemaining = 7
while teachersRemaining >0:
    costVals = np.zeros(9)
    for i in range(0,9):
        newTeacherNum = teacherNum.copy()
        newTeacherNum[i]+=1
        costVals[i] = cost(getRatio(newTeacherNum))
    teacherNum[np.argmin(costVals)] +=1
    teachersRemaining=teachersRemaining-1
print('Ideal number of teachers per department: ', teacherNum)
```

✓ 0.2s

```
Ideal number of teachers per department:  [1 6 4 6 4 7 1 3 6]
```

Solution #2: Complete Search

```
enrollment = np.array([122, 452, 349, 624, 361, 789, 194, 330, 503])
idealRatio = np.array([99, 79.75, 98, 98, 283/3, 647/6, 155, 97, 74.6])
currentTeacherNum = np.array([1, 4, 3, 5, 3, 6, 1, 3, 5])

def getRatio(teachers, studs=enrollment):
    return studs/teachers


def cost(ratio, ideal=idealRatio):
    sum = 0
    for i in range(0, len(ratio)):
        sum += (ratio[i]-ideal[i])**2
    return sum

allottedTeachers = 7
remainingTeachers = allottedTeachers
minCost = cost(getRatio(currentTeacherNum))
minTeach = currentTeacherNum
for a in range(0,8):
    for b in range(0,remainingTeachers-a+1):
        for c in range(0,remainingTeachers-a-b+1):
            for d in range(0,remainingTeachers-a-b-c+1):
                for e in range(0,remainingTeachers-a-b-c-d+1):
                    for f in range(0,remainingTeachers-a-b-c-d-e+1):
                        for g in range(0,remainingTeachers-a-b-c-d-e-f+1):
                            for h in range(0,remainingTeachers-a-b-c-d-e-f-g+1):
                                for i in range(0,remainingTeachers-a-b-c-d-e-f-g-h+1):
                                    teacherNum = currentTeacherNum + np.array([a,b,c,d,e,f,g,h,i])
                                    ratio = getRatio(teacherNum)
                                    cCost=cost(ratio)
                                    if cCost < minCost:
                                        minCost = cCost
                                        minTeach = teacherNum


print('Ideal number of teachers per department: ', minTeach)
```

✓ ✓ 0.9s

```
Ideal number of teachers per department:  [1 6 4 6 4 7 1 3 6]
```

A Consensus: the Best Subjects to Hire Teachers Under

```
Ideal number of teachers per department:  [1 6 4 6 4 7 1 3 6]
```

Department	Current Teachers	Ideal Teachers	New Teachers
Art	1	1	0
Biology	4	6	2
Chemisty	3	4	1
English	5	6	1
Lang	3	4	1
Mathematics	6	7	1
Music	1	1	0
Physics	3	3	0
Social Studies	5	6	1

We should hire:

- 2 biology teachers
- 1 chemistry teacher
- 1 english teacher
- 1 language teacher
- 1 math teacher
- 1 social studies teacher

Updated 2023-2024 Student-Teacher Ratio

Epsilon School 2023-2024 Student Teacher Ratios			
Department	Total Students	Number of Teachers	Student:Teach Ratio
Art	122	1	122
Biology	425	4	106.25
Chemisty	349	3	116.3333333
English	624	5	124.8
Lang	361	3	120.3333333
Mathematics	789	6	131.5
Music	194	1	194
Physics	330	3	110
Social Studies	503	5	100.6

The new teacher quantity fits the 2022-2023 student-teacher ratios significantly better.

Original 2023-2024 Cost: 2656.659

Updated 2023-2024 Cost: 6308.469

Updated Epsilon School 2023-2024 Student Teacher Ratios			
Department	Total Students	Number of Teachers	Student:Teach Ratio
Art	122	1	122
Biology	425	6	70.83333333
Chemisty	349	4	87.25
English	624	6	104
Lang	361	4	90.25
Mathematics	789	7	112.7142857
Music	194	1	194
Physics	330	3	110
Social Studies	503	6	83.83333333

Epsilon School 2022-2023 Student Teacher Ratios			
Department	Total	Number of Teachers	Stud:Teach Ratio
Art	99	1	99
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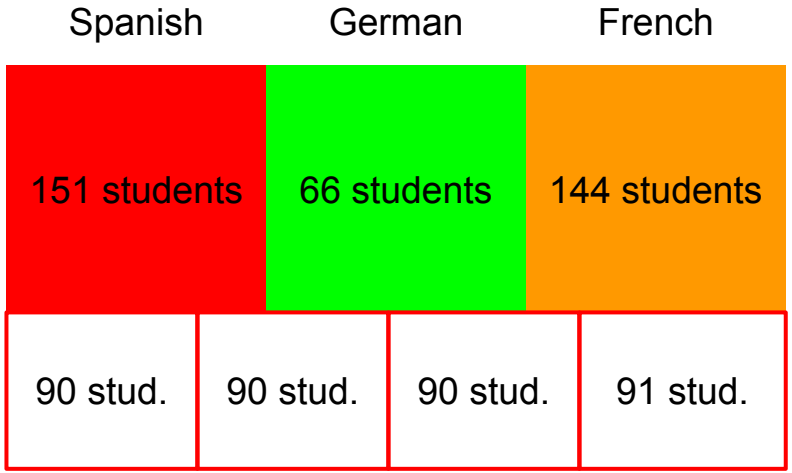
What about the Language Teachers?

361 students, 4 teachers, 3 languages

90.25 students per teacher, or 3 teachers with

90 students, 1 teacher with 91

- 1 pure Spanish teacher w/90 students
- 1 Spanish-German teacher who teaches 61 Spanish and 29 German students
- 1 German-French teacher who teaches 37 German and 53 French students
- 1 pure French teacher w/91 students



Assumptions

- Every student takes one English class
- The percentage of students dropping out each year would be equivalent
- The problem description states that the student population “can be increased from 490 to 630”. We assumed that obtaining the value of 630 is not a necessity.
- For each grade, the percentages of students in each class are roughly the same every year.
- The student-teacher ratios from before the addition of students are the best/ideal ratios
- The three Lang teachers during the 2022-2023 year are shared among the classes & can teach multiple languages

Strengths	Weaknesses
- Adaptable to multiple student or teacher quantities - Light computational requirements - Answers align with other, more thorough methods - able to check/validate answers	- Decreasing number of students per year - Equivalent drop-out rates per year - Every student takes English - Languages are treated as one subject

Future work + acknowledgments

If this project were to continue, we could:

- Try to model the problem for other years (such as 2024-2025) to see if the hired teachers are still ideal.
- Different dropout rates (other than 5%)
- Because of the dropout rate, the student population is decreasing year by year. Finding how to solve this decrease would be an area of interest
- Treat foreign language courses separately

We would like to acknowledge the classmates that helped us during XYZ groups and their useful ideas.

Thank you for listening

Questions?

Works Cited

- https://learn.org/articles/massachusetts_high_school_diploma_requirements.html
- https://easyai.tech/en/ai-definition/gradient-descent/