Question: In a modified Atwood machine where the track is at an incline (Fig 1), how does the angle of the track (θ) affect the acceleration of the system, assuming no friction?

Hypothesis: The relationship between the track angle and acceleration is sinusoidal, and increasing the angle will decrease the acceleration (assuming the angle is between 0° and 90°).

Strategy:

- One end of the track was kept on the ground, while the other end was raised by a pile of books (the blue boxes in Fig 1). The angle of the track was varied by changing the height of the track, which was done by decreasing the number of books in the pile.
- At each angle (θ), the resulting acceleration was measured three times using a Vernier motion detector, and an average was taken.
- The mass on the cart (m_1) and the hanging mass (m_2) was kept constant.
- The angle (θ) was graphed vs. the measured acceleration, and using sinusoidal regression the "sinusoidal curve of best-fit" was found. The equation of this curve was then compared to the true equation found by using Newton's Second Law, verifying it.

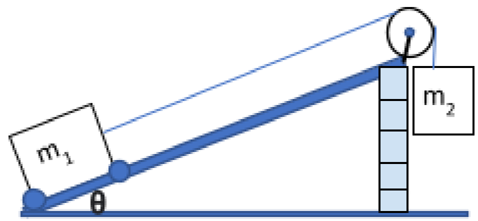


Fig 1: Inclined Modified Atwood's machine

Data:

Mass of cart → $m_1 = 0.5018\ kg$.

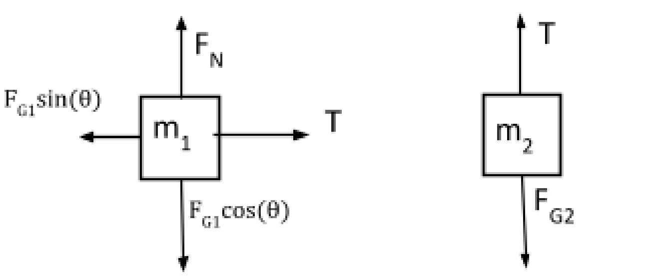
Hanging mass → $m_2 = 0.1999\ kg$.

#Books	Angle (degrees)	Avg. Acceleration (m/s²)
0	0	2.729333333
3	3.5281	2.340666667
6	6.8862	1.955666667
9	9.8339	1.623666667
12	13.0317	1.193666667

The acceleration is an average of three trials

Analysis:

The free body diagrams below show the forces on the masses in the inclined modified Atwood's machines. For the cart mass (m_1), the forces in the diagram are relative to the track:



We're assuming there's no friction between the cart and track because the wheels spin freely. The following equations are based on the free body diagrams. Positive motion is defined as to the right for the cart, and down for the hanging mass:

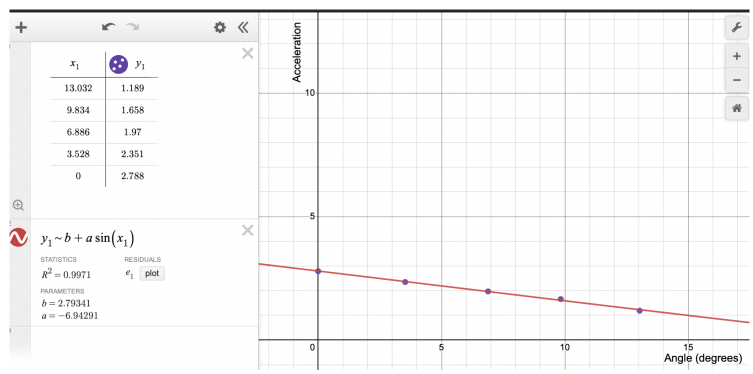
$$T - m_1 g sin(\theta) = m_1 a$$

$$m_2 g - T = m_2 a$$

Adding these two equations and isolating a:
$a = gm_2/(m_1 + m_2) - gm_1/(m_1 + m_2)sin(\theta)$
With our given values for m_1 and m_2, this is just:

$a = 2.7918 - 7.0082 sin(\theta)$

Therefore, in theory, the *track angle has a sinusoidal relationship with acceleration*, one of the form $y = a + bsin(x)$. Looking at the data, we can plot the graph of theta vs acceleration and run sinusoidal regression in DESMOS to see how well it fits.



In the diagram above, the purple points represent our data. It's clear from the graph that *increasing the angle decreases the acceleration.* The red line represents the "sinusoidal curve of best fit, and it's equation is:

$a = 2.79341 - 6.94291 sin(\theta)$

This curve fits the data very well (the R-value is approximately 0.9985), which verifies that the relationship between track angle and acceleration was indeed sinusoidal. Furthermore, the equation of the curve-of-best fit is very close to the theoretical equation found earlier. Specifically, the y-intercept was only 0.058% larger than expected, and the coefficient of $sin(\theta)$ was 0.932% less than expected. The discrepancy in the y-intercept value was most-likely due to a measuring error, since the difference was so minimal. The coefficient of $sin(\theta)$ was less than expected, which could be a result of friction between the wheels and the cart. This would have slightly decreased the acceleration, thereby decreasing the coefficient.