Problem A : The Need for Bees (and not just for honey)

1. Summary

Colony collapse disorder (CCD) in bee colonies poses a major threat to humanity because of the integral benefits bees provide in the pollination cycle (index : USDA ARS, n.d.). Viruses, pesticides, predators, habitat destruction, and environmental conditions are the main contributors to the issue. However, understanding the trends of bee colonies in general is largely unknown, meaning that this ongoing issue may be further exacerbated without models. Therefore, an accurate model is a priority for society as a whole. At first glance, creating a basic regression model with facts included seems plausible. However, the complexity and broadness of the issue become evident because of factors such as season and queen egg laying rate. Furthermore, when looking at the problem from a factor-level, it becomes clear that this view is of especial importance as it allows for sensitivity testing which can be utilized to better mitigate the issue. The only pieces that impact the population of a colony are the births per set time period and the deaths per set time period, but it is difficult to model the individual pieces that influence each. In this paper, we develop two models to depict the change in population of honeybee colonies as a function of time. We consider both a simple approach to the problem, involving non-complex parameters from the problem statement as well as a complex approach that takes nearly every variable impacting a honeybee colony into account. From there, we compare our solutions to pre-existing solutions and interpret the impact based on our models.

In order to create a simple model, only birth and death rates in terms of time were observed and modeled. The factors observed were strictly eggling rate, lifespan, level of activity, availability of food, protein levels, and nectar values. Each of these variables dynamically changed based on several subfactors. Therefore, recursion was utilized to provide a comprehensive and robust model.

Though the first model provides a general direction of the trend for a colony population, accuracy can be improved by considering more factors and making the pre-existing factors more realistic. With more factors considered such as mites (Diseases of Bees, n.d.) or rates of survival for eggs, larvae, and pupa, the model becomes closer to modeling a real-life colony. A previously known model entitled “HoPoMo” demonstrated the trend of bee population on the basis of 20+ factors; however, the authors of this study stated deficiencies and missing parameters from the model (Schmickl & Crailsheim, 2007). Therefore we built onto factors stated as missing, insufficient, or unrealistic. In this sense, model 2 extends beyond simply the scope of the problem statement and serves to improve the accuracy of pre-existing work to further models in the realm of CCD.

Even with both of these models, CCD can only be addressed by determining which specific factor has the most impact on our model. For instance, if it is concluded that available food is the greatest issue for honeybee colony sustainability based on testing, future research can be pointed to looking at creating safe breeding environments for honeybees where there is enough food to live. Thus, sustainability testing serves to be an important factor in order for this paper to provide the most context for future works. We planned to do so by utilizing SQL queries to existing databases as well as statistical comparative analysis in Python 3.0.1.

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2. Blog

The Bee Daily: The Need for Bees

(Bernardini, 2022)

Over the course of the past few years, several issues ranging from tsunamis to COVID-19 have taken place, yet in light of these events, one has gone unnoticed—colony collapse disorder (CCD). This issue arose in light of the decline of the honeybee population. These populations are severe as an essential component to life on earth because of their pollination abilities. However, as of 2007, the term CCD was coined in order to raise attention for this growing trend. As a result, anonymous highschoolers with the name “Team 12157” both attempted and crafted two comprehensive models to aid mathematicians and scientists in future research of the issue. The models make use of recursion, python scripts, and past research.

Knowing that skill levels in mathematics, the team created a simplified model and a complex model to aid users. The simple model takes only two ideas into account: bee population rests on day-to-day births and deaths. Therefore, this model becomes heavily dynamics-based, involving the use of calculus to create a recursive model that can be computed by the everyday person. To contrast this idea, the complex model builds off past research. The team conducted several sleepless nights of research to find holes in an existing model of bee population modeling. They investigated each of the holes in the model, solving complex dynamic equations that could then be re-integrated with the original model to provide a more accurate model both in scope of this problem statement as well as for future works.

Though these models appear to be highly cohesive and important for decision makers, they are effectively unimportant to the everyday person until the significance is considered. According to analysis from the model, after 48 days an average bee colony will completely collapse. So what’s the issue? Create another colony, right? Unfortunately, when these results are generalized across a greater population of colonies. If this trend continues, a significant amount of bees will die before the year 2050. However, to combat these potentially catastrophic impacts, the team also conducted a sensitivity analysis on the models. They found that survivability rate was the biggest contributor to increasing bee deaths. In order to solve this issue, the team hypothesized that bees would be better protected to live longer in the future. Although the issue would not be fully solved, by addressing this specific causation factor within the larger problem, the team predicts that 45% of honey bee deaths will occur. Finding this adjusted rate was crucial for the team as it enables experts to understand the extended timeline they may have to solve the larger issue if smaller issues such as survivability are prioritized first.

Thus, thanks to the comprehensive work of team 12157, two full models have been created in addition to sensitivity testing that can alert experts as well as a solution to the amount of bee hives required to support 20 acres of land.

3. Problem Introduction

3.1 Initial Questions & Problem Analysis

Problem A of the 2022 HiMCM math competition—the Need for Bees (and not just for honey)—requires a model for the decline of bees based on real-world factors. When creating this model, viruses, pesticides, predators, habitat destruction, and environmental conditions need to be included in order to accurately depict the ongoing situation of colony collapse disorder (CCD). Furthermore, several pieces of information were provided by the problem. Honeybees have the ability to travel up to 20 km but tend to stay at 6km. Additionally, honeybees can visit 2,000 flowers or more each day. Beyond these statistics, information about seasons was provided. Honeybees work more during the summertime and therefore have higher rates of death. On the contrary, during the autumn and wintertime, honeybees tend to live longer because of the lowered amount of work. Finally, levels of activity, pollen consumption, and protein abundance are also given as variables that impact a bee's lifespan. In order to provide general readers understanding of our model, we were additionally asked to create a non-technical blog. With this information provided, we asked a series of questions to understand and address our approach to the problem:

1. What are all of the factors that must be considered when analyzing the lifespan of a bee
2. On a day-to-day basis, how many honeybees are born and how many die? What factors contribute towards this?
3. How are the death of bees interconnected with the birth of bees?
4. Does age demographic impact the model?
5. What types of honeybees should be focused on, if any?
6. How do seasons impact the honey bee population?
7. How could external factors such as humans and weathering be incorporated into the population model?

3.2 Mind Mapping

Prior to attempting to model, our team decided to map our ideas based on the information purely given by the problem. In order to do so, we created a mindmap on Miro, providing directionality to our model. Figure 1a depicts our ideas. We decided that there must be two main factors to determine the population of a colony: births and deaths. Each of these functions contributing to population change was modeled with parameter t to demonstrate that the main causation variable will be time.

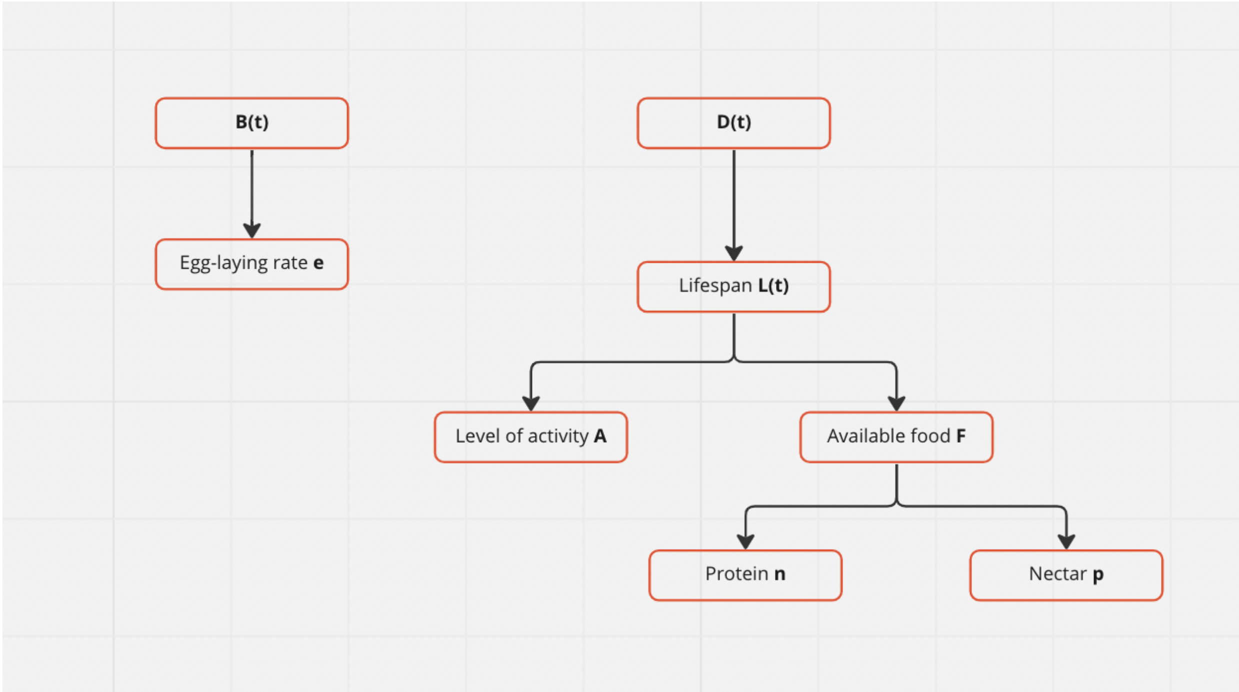


Figure 1a. Initial Brainstorming

3.3 Data

No datasets were provided by the problem statement. However, in order to sensitivity test models, population sizes would be required. Therefore we randomly generated numbers from 20,000 to 80,000 as a dataset for future use. In order to test sensitivity, Python was utilized to expedite the process. Because of this use of technology, 100,000 trials were conducted and the averages of each parameter were weighted based on results. The code to find these random numbers is provided in appendix 6.2. Furthermore, the scatterplot below demonstrates the distribution of random testing values.

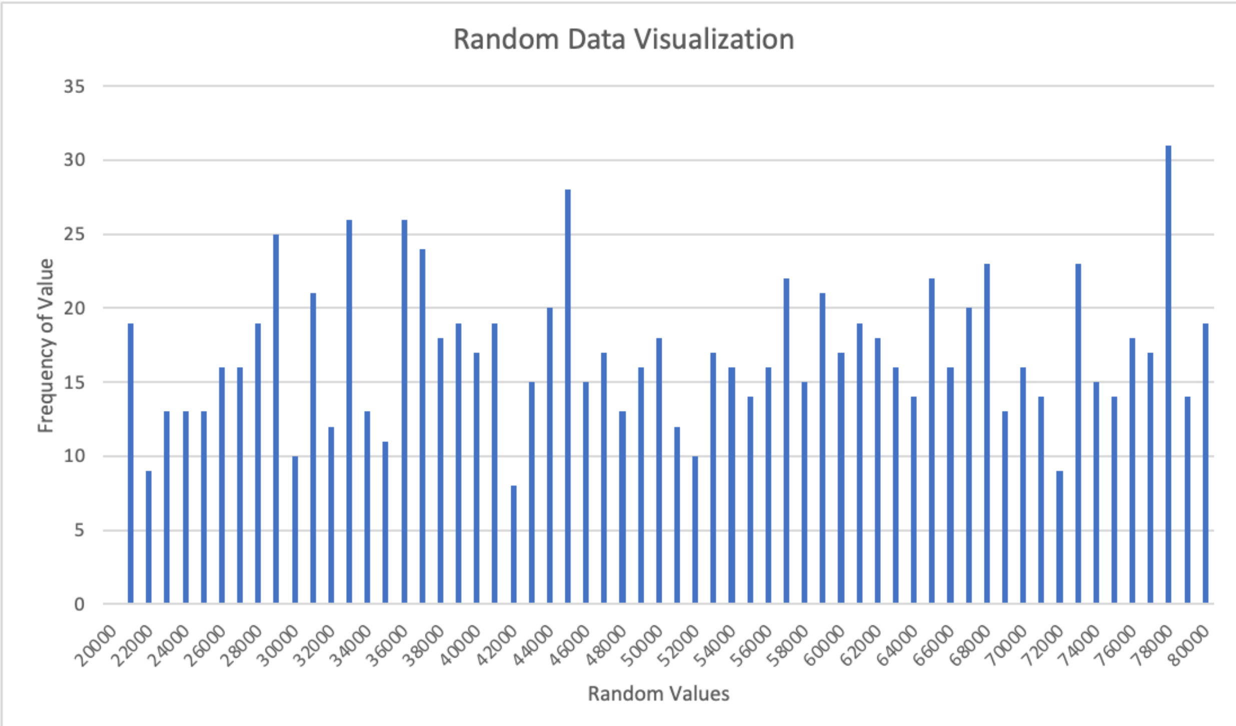


Figure 1b. Testing Data

Beyond sensitivity testing, the models needed to be tested against real-world data to demonstrate the reliability of the models. Utilizing a database from Data.World entitled “2018/W18: Bee Colony Loss” by Evan Murray, SQL queries shown in appendix _ were utilized to output the average percent loss (). Our queries yielded an average percent decrease in colony populations of 0.41144 for 1 year. Because these data values come from US means, they can be compared to percent decrease for one year of our model to demonstrate reliability. A sample of the data used is demonstrated below:

	year	season	state	total_annual_loss	beekeepers
1	2016/17	Annual	Massachusetts	0.159	87
2	2016/17	Annual	Montana	0.171	21
3	2016/17	Annual	Nevada	0.23	13
4	2016/17	Annual	Maine	0.233	65
5	2016/17	Annual	Wyoming	0.234	18
6	2016/17	Annual	Hawaii	0.262	10
7	2016/17	Annual	Mississippi	0.263	9

1c. Sample Data

4. Assumptions

1. The honeybees being referenced to are the Eastern or Western honeybees

Justification: Western and eastern honey bees rank as one of the main pollinators (What's Happening to the Bees? - Part 5: Is There a Difference between Domesticated and Feral Bees?, 2014)

2. The survival rate for honeybees are constant in their respective castes.

   Justification: Each stage of the honeybee faces similar risks ranging from 9% to 36% in each caste (Prado et al., 2020).

3. The mortality rate every day for any stage of bee birth is the same

   Justification: Western honey bees rank as one of the main pollinators

4. Assumed that seasonal effects in the northern hemisphere provide a fair overview of overall effects

   Justification: Impacts such as the decrease due to winter season can only be attributed to colder climates, therefore the northern hemisphere is an important climate region to be considered (Donkersley et al., 2017)

5. The survival rate of the forager bee caste will change throughout the colony lifespan

Justification: Survival rate of the bee will change due to the constant change in lifespan of the bees from season-to-season.

5. Model 1: Recursive population model

5.1 Componentizing the Bee Colony

From 3.1, it becomes clear that the most important aspect in solving our model comes with figuring out the equations that express births per *t* and deaths per *t*. In order to determine the population of bees living in a colony, the different factors in each of these categories need to be broken down and considered. In this sense, we are componentizing the colony with a series of equations that aid our final model. Under the category of births—B(t)—we only consider eggling rate. Studies have shown that eggling rate proves to be the most important factor in impacting the population (Burlew, 2022). In order to calculate eggling rate, we understood that several factors would be involved such as the health of the queen and seasonal conditions. Using

5.2 Methodology

This model determines the population of a honeybee colony recursively. We let $P(t)$ represent the total population of colony at a time t, given in days. $P(t)$ can be split up as the sum of the total numbers of bees in each caste. These are five bee castes which we consider:

- Eggs $\longrightarrow T_e(t)$ is the total number of eggs at time t
- Larvae $\longrightarrow T_l(t)$ is the total number of larvae at time t
- Pupae $\longrightarrow T_p(t)$ is the total number of pupae at time t
- Nurse bees $\longrightarrow T_n(t)$ is the total number of nurse bees at time t
- Forager bees $\longrightarrow T_f(t)$ is the total number of forager bees at time t

Therefore, we can write $P(t)$ as:

$$P(t) = T_e(t) + T_l(t) + T_p(t) + T_n(t) + T_f(t)$$

Therefore, to find $P(t)$ we can look at each part individually. We can look at $T_e(t)$. Below are the variables/functions which affect $T_e(t)$:

- $E(a)$ is the number of eggs that are of age a
- S_e is the survival rate of eggs per day
- $B(t)$ is the birth rate of the queen bee at time t
- $T_e(t)$ is the total number of eggs at time t
- u_e is how many years a bee stays as an egg before it turns into a larvae

At any time, we let $E(a)$ be the number of eggs that are of age a. Therefore, the total number of eggs at a given time would be the integral of $E(a)$ from 0 to u_e, where u_e is the maximum age of an egg before it hatches into a larvae:

$$T_e(t) = \int_0^{u_e} E(a)\, da$$

However, since the birth rate of the queen is not constant, the shape of $E(a)$ changes as the birth rate $B(t)$ changes. If the birth rate changes, then the young eggs are impacted first. Thus, it is better to approximate $E(a)$ by splitting up the integral into smaller integrals with an equal width of Δ. The picture below shows the integral partitioned into 13 parts, P_0 to P_{12}, all with width Δ. This can be generalized:

$$T_e(t) = \int_0^{u_e} E(a)\,da = \sum_{i=0}^{\frac{u_e}{\Delta}-1} P_i = \sum_{i=0}^{\frac{u_e}{\Delta}-1} \int_{i\Delta}^{(i+1)\Delta} E(a)\,da$$

Now consider how this changes when Δ days elapse. We assume that the survival rate of an egg S_e is constant. After Δ days, out of the bees that were k days old, only S_e^{Δ} of them survived and are now $k+\Delta$ days old. Thus, $P_k = (S_e^{\Delta})P_{k-\Delta}$. From all the parts in $T_e(t)$, all except the last one remain eggs. Therefore, the number of eggs that survived and stayed eggs after Δ days is:

$$\text{Total parts} - \text{Last part} = S_e^{\Delta} T_e(t) - \text{Last Part}.$$

To find the value of the last part (P_{12} in our example), we use the fact that they were born $u_e - \Delta$ days ago. The number of eggs that were born at that time was $B(t-u_e+\Delta)$, so $E(a) = B(t-u_e+\Delta) * (S_e)^a$. To find the last part this can be integrated, which gives us a value of $B(t-u_e+\Delta) * (S_e^{\Delta+u_e} - S_e^{u_e})$.

We have taken in consideration the eggs that survived after Δ, but we did not consider the eggs that were recently born. This is:

$$\int_0^{\Delta} B(t+\Delta) * (S_e)^a\,da = B(t+\Delta)(S_e^{\Delta} - 1)$$

This can be done similarly for the larvae:

$$T_l(t+\Delta) = S_l^{\Delta} T_l(t) + B(t-u_e+\Delta) * (S_e^{\Delta+u_e} - S_e^{u_e}) - B(t-u_e-u_l+\Delta) * (S_e^{\Delta+u_e+u_l} - S_e^{u_e+u_l})$$

and pupae:

$$T_p(t+\Delta) = S_p^{\Delta} T_p(t) + B(t-u_e-u_l+\Delta) * (S_e^{\Delta+u_e+u_l} - S_e^{u_e+u_l}) - B(t-u_e-u_l-u_p+\Delta) * (S_e^{\Delta+u_e+u_l+u_p} - S_e^{u_e+u_l+u_p})$$

and nurse bees:

$$T_n(t+\Delta) = S_n^{\Delta} T_n(t) + B(t-u_e-u_l-u_p+\Delta) * (S_e^{\Delta+u_e+u_l+u_p} - S_e^{u_e+u_l+u_p}) - B(t-u_e-u_l-u_p-u_n+\Delta) * (S_e^{\Delta+u_e+u_l+u_p+u_n} - S_e^{u_e+u_l+u_p+u_n})$$

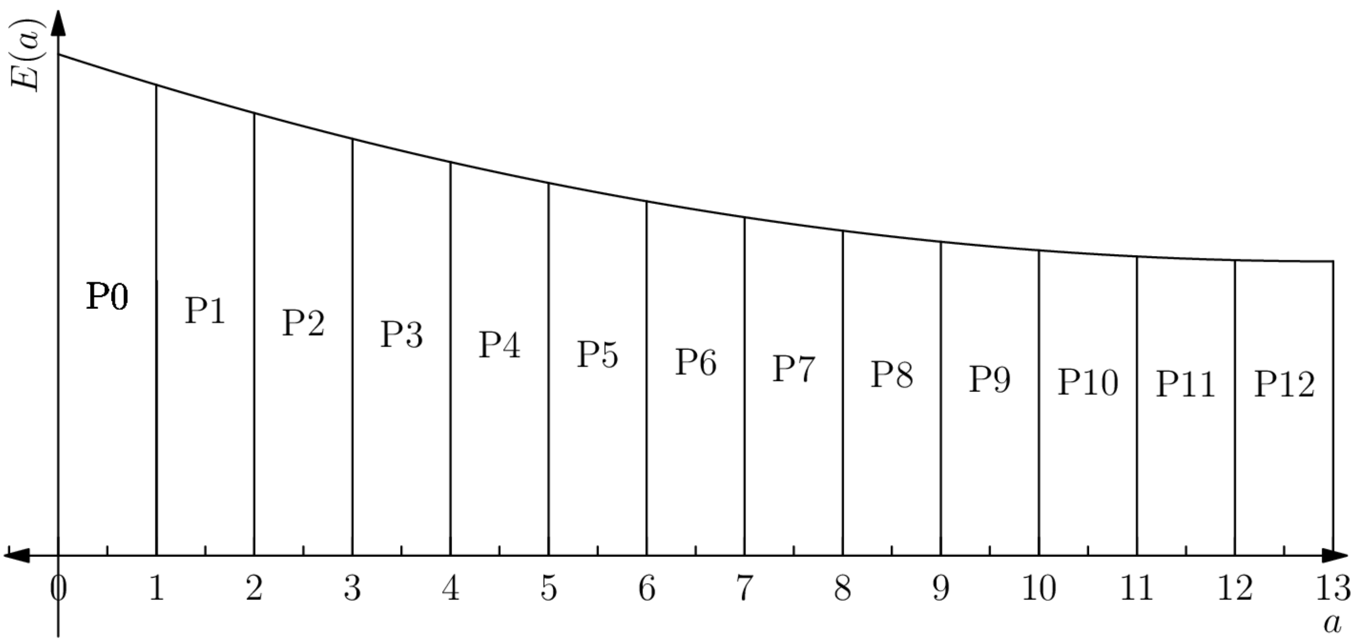
and finally for forager bees:

$$T_f(t+\Delta) = S_f^{\Delta} T_f(t) + B(t-u_e-u_l-u_p-u_n+\Delta) * (S_e^{\Delta+u_e+u_l+u_p+u_n} - S_e^{u_e+u_l+u_p+u_n}) -$$

$$B(t - u_e - u_l - u_p - u_n - u_f + \Delta) * (S_e^{\Delta + u_e + u_l + u_p + u_n + u_f} - S_e^{u_e + u_l + u_p + u_n + u_f})$$

In total, the five recursive equations for T_e, T_l, T_p, T_n and T_f can be used to find the total populations at any given time, and can be as precise as you want by setting Δ closer to 0.

The above explanation was made in Latex.



(Picture demonstrating E(a) partitioned)

6. Model 2: Modification of a Steady Model

Model 2 is a model that builds upon and improves a preexisting model from the paper “Modeling Honey Bee Populations” by David J. Torres, Ulises M. Ricoy, and Shanae Roybal. This pre-established model was concise and reliable, but it failed to consider several important aspects. First, it assumes a constant egg laying rate(ELR). In actuality, the number of eggs the queen lays each day can be varied by a multitude of different factors, like the weather, season, amount of resources, age, etc. Next, the survival rate and lifespan for the forager bees should not be considered as a constant like in the study. For example, survival rate and lifespan are much shorter during the summer than in the winter, likely due to the cause of over exhaustion. Lastly, the article does not take time into account at all, leading to the poor predictions across long periods. These 3 directions where we focused on improving the model.

6.1 Original Model

The original model considers a colony of bees under 5 main castes, the eggs(E), larvae(L), Pupae(P), drone and nurse bees under hives(H), and foragers(F). E_0 is the eggs laid every day. S_{egg}, S_{larvae}, S_{pupae}, S_{hive}, and $S_{forager}$ represent the daily survival rate of each of the castes, and the number of days spent in each bee caste is represented by n_{egg}, n_{larvae}, n_{pupae}, n_{hive}, and $n_{forager}$.

Let's turn our attention to the first part, finding the E, or the number of eggs.

$$E = E_0 \sum_{i=0}^{n_{egg}-1} S_{egg}^{i} = E_0 \frac{1 - S_{egg}^{n_{egg}}}{1 - S_{egg}},$$

A summation is used to represent the total number of eggs. The amount laid per day, E_0, is multiplied by the daily survival rate once for every day that passes since. This is stopped after the eggs goes on to the next stage of larvaes after n_{egg} days. All the other castes are similar to consider, except that instead of multiplying the summation by E_0, it is multiplied by the amount of original eggs that has survived to that point.

Finally,

$$L = E_0 S_{egg}^{n_{egg}} \sum_{i=0}^{n_{larvae}-1} S_{larvae}^{i} = E_0 S_{egg}^{n_{egg}} \frac{1 - S_{larvae}^{n_{larvae}}}{1 - S_{larvae}},$$

$$P = E_0 S_{egg}^{n_{egg}} S_{larvae}^{n_{larvae}} \sum_{i=0}^{n_{pupae}-1} S_{pupae}^{i} = E_0 S_{egg}^{n_{egg}} S_{larvae}^{n_{larvae}} \frac{1 - S_{pupae}^{n_{pupae}}}{1 - S_{pupae}},$$

$$H = E_0 S_{egg}^{n_{egg}} S_{larvae}^{n_{larvae}} S_{pupae}^{n_{pupae}} \sum_{i=0}^{n_{hive}-1} S_{hive}^{i} = E_0 S_{egg}^{n_{egg}} S_{larvae}^{n_{larvae}} S_{pupae}^{n_{pupae}} \frac{1 - S_{hive}^{n_{hive}}}{1 - S_{hive}},$$

$$F = E_0 S^* \sum_{i=0}^{n_{forager}-1} S_{forager}^{i} = E_0 S^* \frac{1 - S_{forager}^{n_{forager}}}{1 - S_{forager}}$$

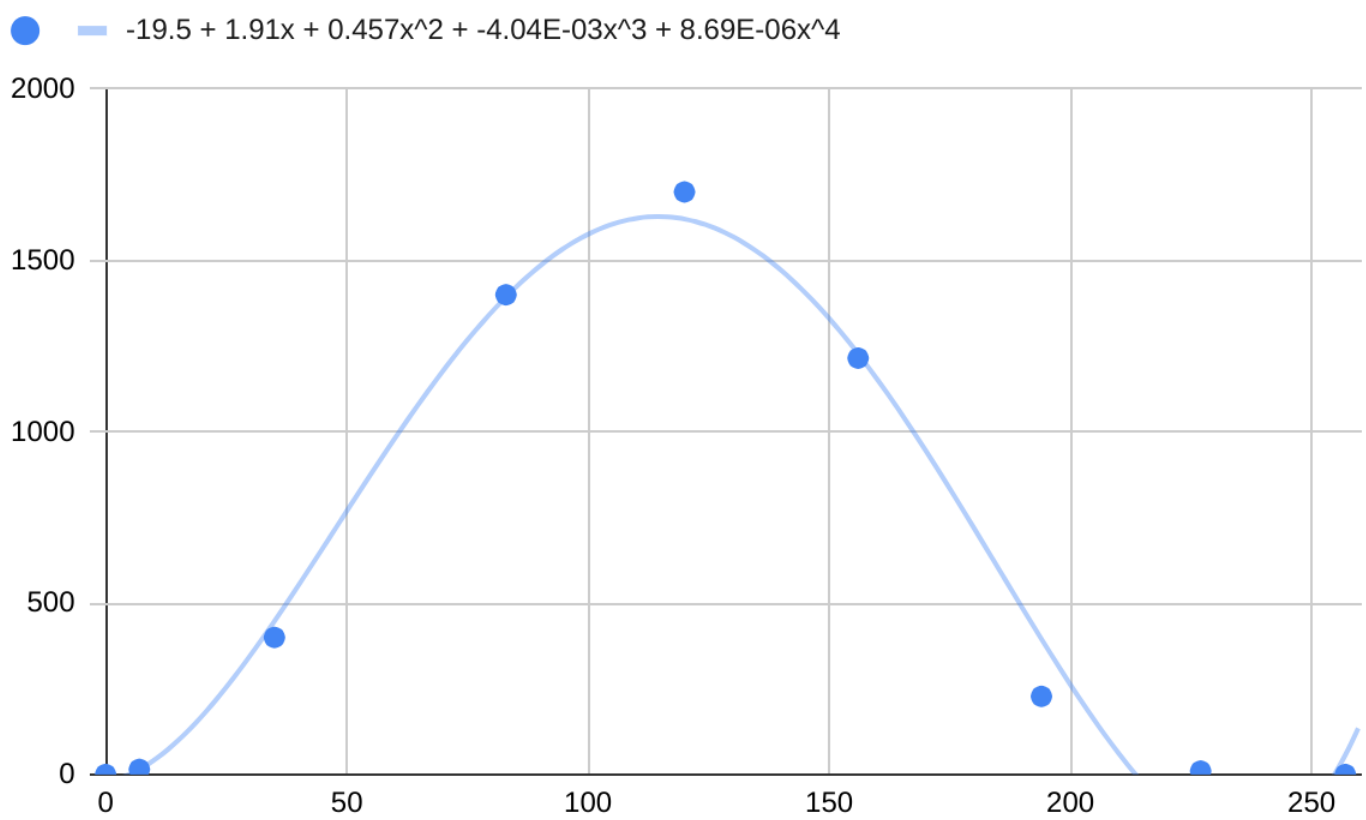
$$T = E + L + P + H + F.$$

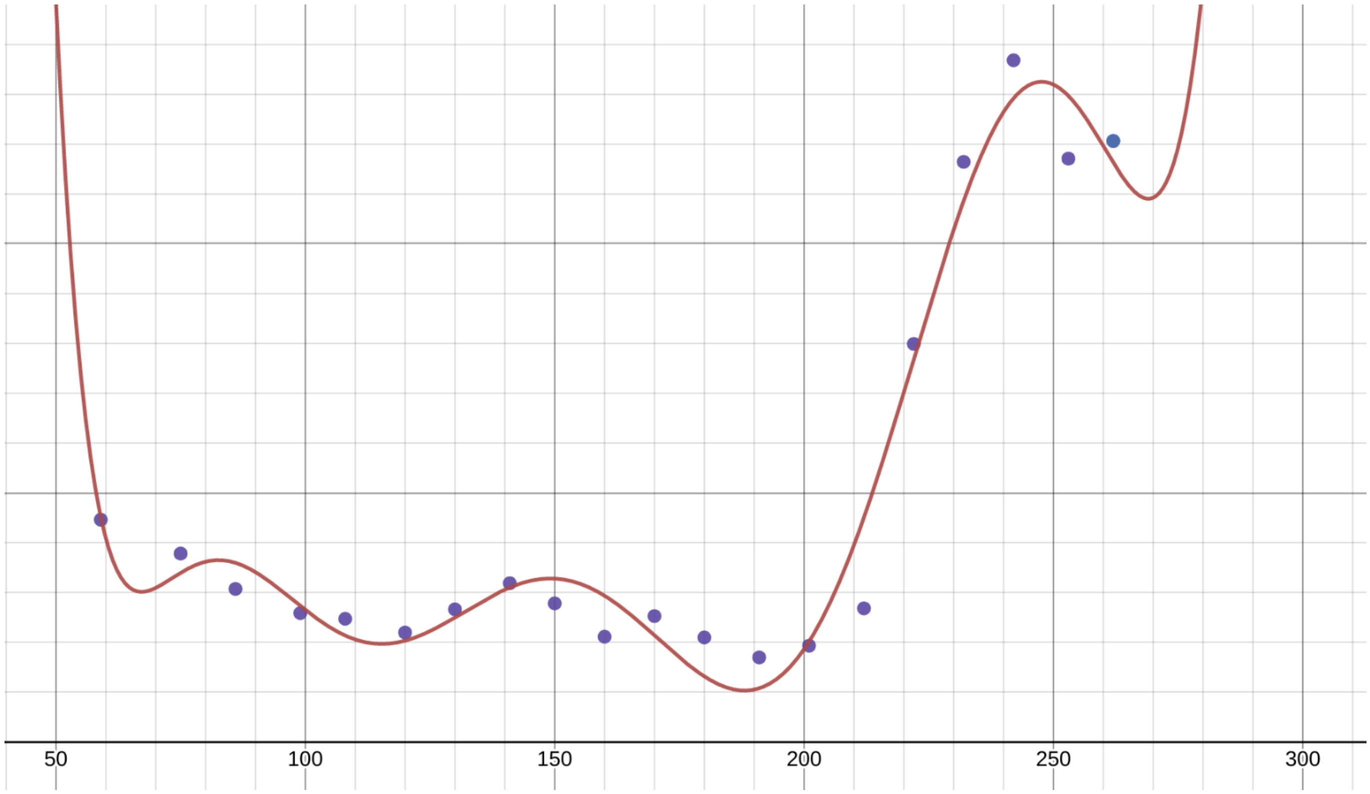
(Schmickl et al., 2007)

6.2 Improvements

In order to improve the model, the equation to calculate the number of foragers has to be improved. $N_{forager}$ and $S_{forager}$ were modeled as functions N(t) and S(t) where t is the number of days after february 1st to more accurately represent the trends that happen within a year.

$$n(t) = \sum_{i=0}^{\infty} ((1 - s(i))(i + 1) \prod_{x=0}^{x=i} s(i)$$




50
100
150
200
250
300

7. Sensitivity Testing

In order to test sensitivity, a Python script was utilized to run 100,000 trials with the data from section 3.3 on model 1 to start. After our trials, it was demonstrated that the survivability rate had a greater impact on the population compared to egg laying rate. Thus, in terms of only this model, survivability rates must be improved in order to create a more sustainable colony. From a non-mathematical standpoint, this conclusion seems plausible because the survival rate also takes into account the birth of eggs. Therefore, in this sense, survivability is integrated in both the death and birth aspects of the model to an extent. In comparison, the egg laying rate is only integrated into births. For these reasons, the rate of survivability becomes key to improve. The rate of survivability breaks into numerous factors. This figure accounts for the rate of death that occurs for worker bees in the field, disease, and more. In comparison, egg laying rate was modeled by a function called ELR from past models. This rate is much more simplified and only looks at birth rate in terms of time.

8. Hives Required to Sustain 20 Acres

The number of hives that would be required to pollinate a 20 acre parcel of land is highly dependent on several factors such as the weathering conditions, the availability of crops in that area, and the pesticides.

8.1: Amount of Foraging

Studies have shown that temperature has a direct correlation on worker bees and how much foraging (pollination) they do (*Washington State University)*.

Temperature	Foraging
65°F	100%
63°F	62%
54°F	21%
51°F	6%

(*Washington State University)*

From the given information, we were provided that each worker bee can travel 20 km but tend to stay within 6 km of their hive to ease the workload. In addition, it was provided that a bee visits approximately 2000 flowers a day.

The primary approach was to first figure out how many available crops there would be for the bees to pollinate in the provided space, in this case 20 acres. Based on our developed population models, we aimed to figure out the number of workers that would be pollinating and see how many crops one hive would be able to pollinate. Based on the studies, it was researched that there are an estimated 100 trees per acre, and 20,000 flowers per tree (*Almond pollination math)*. Therefore 100*20,000*20 = 40,000,000 million flowers need to be pollinated in a day. Although a worker bee is capable of doing 2000 flowers in a day, that number is largely dependent on what temperature it is. And referring back to the assumptions list, it was inherited that this colony and land is located in a colder type climate in order to accommodate and model the impact of seasonal factors. With the average temperature in the northern hemisphere around 59.62 Fahrenheit, only about 400 flowers would be pollinated, therefore 40,000,000/400 requires 100,000 bees (*Climate Spatial Scales)*. Based on our calculated model, and the population as a function of time, the number of hives could differ at a given moment, but with an average colony starting at 50,000 bees and decreasing, the number of hives required would be **10 hives.**

9. Conclusion

Strengths

One of the strengths of our solution was the creation of two different models. The use of multiple models allowed us to have a greater chance of achieving higher accuracy. Furthermore, these different models utilized different concepts to achieve the solution. Therefore, in this sense, the drawbacks of one concept are accounted for by the strengths of the other. Additionally, the use of code and existing databases allowed us to have greater confidence in our end results.

Weaknesses

The first weakness comes because of our testing strategy. In creating the random values we use pseudorandom values which are not completely random. Therefore, for the second part of the solution involving sensitivity analysis, the results may have been skewed by this lack of randomness. However, pseudorandom models do still provide some aspect of randomness, just not true randomness. Furthermore, model two did not reach a complete conclusion and therefore cannot fully be used. However, with more time, the model would have been completed and would have been able to provide more context on pre-existing works. Though, model 1 did still fully function and work. In addition, because of insufficiency with model 2, question 3 was solved in an inaccurate method that may produce a flawed answer.

Significance

Our results will help researchers in the future better model bee populations counter these issues. For instance, the first model takes into account two main factors which can be tested for and the resulting values can be used to make better assessment in the future. Additionally, we were able to improve an existing model which may aid future work. Furthermore, our models are aimed at solving an important and growing issue that has the potential to negatively impact many.

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11. Appendix

6.1 Python Random Number Generation

```
import random
randList = []
for i in range(100000):
    val = random.randrange(20000, 80001)
    randList.append(val)
    print(val)
```

6.2 Database SQL Queries

```
SELECT AVG(total_annual_loss)
FROM bee_colony_loss
```